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# Dilaton as the Higgs boson

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We propose a model where the role of the electroweak Higgs field is played by the dilaton. The model contains terms which explicitly violate gauge invariance, however it is shown that this violation is fictitious, so that the model is a consistent low energy effective theory. In the simplest version of the idea the resulting low energy effective theory is the same as the top mode standard model.

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**Introduction** Fundamental vector fields are the carriers of the strong, weak and electromagnetic interactions of the Standard Model. A consistent quantum mechanical description of these vector fields (massless and massive) is possible within the context of locally gauge invariant theories. More specifically the massive weak bosons  $W_\mu^\pm$  and  $Z_\mu$  are treated as the gauge bosons of a spontaneously broken  $SU(2) \times U(1)_Y$  electroweak gauge symmetry [1], which is simply realized through the Higgs mechanism [2]. The theory predicts the existence of an electrically neutral scalar field called the Higgs boson. The Higgs boson interacts with other Standard Model particles, notably with the weak gauge bosons, with couplings dictated by gauge invariance. As a result, scattering amplitudes of the longitudinal modes of massive weak bosons satisfy the unitarity bound, provided the Higgs boson is not too heavy,  $m_h < 1$  TeV or so.

The dilaton field is motivated in many extensions of the Standard Model which attempt to consistently incorporate gravitational interactions. In fact many known extensions of the Einstein theory of gravitation are of the Brans-Dicke type scalar-tensor theories. For example, in string theory the graviton is inevitably accompanied by the dilaton, whilst in models of electroweak symmetry breaking with a near or exact scale invariance the dilaton may play a role in low energy phenomenology (see e.g., [3], [4]). If the dilaton field is indeed present in the particle spectrum, one may ask whether it can be identified with the Higgs boson. This is the question we would like to discuss in the present paper<sup>2</sup>.

At first glance it appears that the dilaton can not play the role of the Higgs boson since it does not couple to the Standard Model fields in a gauge invariant way. Actually, because the Standard Model Lagrangian without the Higgs sector is classically scale-invariant, the dilaton can be completely removed at the classical level by an appropriate conformal transformation. Therefore the scale-invariance of the Standard Model must be broken in order to couple the dilaton to matter fields. In this work we consider models where scale invariance is broken explicitly by mass terms for the fermion fields. These fermion mass terms also explicitly break gauge invariance and thus it seems we achieve nothing. Nevertheless, we show that if the dilaton is a non-dynamical degree of freedom classically, the above explicit breaking of gauge invariance is actually fictitious. The key point is that the system possesses a gauge invariant constraint, induced by the equation of motion for the non-dynamical dilaton field, which enforces full gauge invariance. It turns out that the simplest implementation of this idea reduces to the gauged Nambu-Jona-Lasinio model [6], which allows for dynamical electroweak symmetry breaking where the dilaton plays the role of the Higgs boson.

**The basic idea** Let us start by considering a locally gauge invariant theory with gauge group  $G$ . Besides the gauge fields,  $A_\mu \equiv A_\mu^a T^a$  ( $T^a$  are generators of  $G$ ), we

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<sup>2</sup>See also [5] for another interesting non-standard analysis of the electroweak symmetry breaking sector.

introduce two fermion fields  $\psi(x)$  and  $\chi(x)$ , as well as a non-dynamical real scalar field  $h(x)$ . These fields transform under  $G$  as

$$G: A_\mu \rightarrow UA_\mu U^{-1} + \frac{i}{g}U\partial_\mu U^{-1}, \quad \psi \rightarrow U\psi, \quad \chi \rightarrow \chi \quad \text{and} \quad h \rightarrow h, \quad (1)$$

where  $U(x) \in G$ . The Lagrangian of the theory is written as

$$\mathcal{L} = -\frac{1}{2}\text{Tr}(F_{\mu\nu}F^{\mu\nu}) + i\bar{\psi}\gamma^\mu(D_\mu\psi) + i\bar{\chi}\gamma^\mu(\partial_\mu\chi) + [\lambda\bar{\psi}h\chi + \text{h.c.}], \quad (2)$$

where  $D_\mu \equiv \partial_\mu - igA_\mu$  is the covariant derivative and  $F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$  is the  $G$ -valued field strength. Except for the Yukawa interaction terms in parenthesis, the above Lagrangian is invariant under the local gauge transformation (1). Note also that the kinetic term for  $h$  is absent in (2), i.e.  $h$  is non-dynamical. Because of these apparent drawbacks, the model described by the Lagrangian (2) can not be renormalizable. We assume that it holds at a certain high energy scale  $\Lambda$ . Moreover, since the gauge invariance is broken *explicitly* one might expect violation of unitarity in, e.g., the scattering amplitudes of longitudinal gauge field modes. The latter expectation is wrong however. It turns out that, due to the non-dynamical nature of the scalar field  $h$ , the explicit breaking of gauge invariance is fictitious in the sense that the Lagrangian (2) actually describes gauge invariant dynamics. To see this we rewrite (2) in an equivalent form with manifest gauge invariance.

The fact that (2) describes gauge invariant dynamics is far from obvious. Indeed with the field variables used in (2) it is impossible to solve in closed form all the constraints which enforce the gauge invariant dynamics. Fortunately, we can introduce new field variables such that the gauge invariant constraints can be solved explicitly. To this end, let us consider the unimodular scalar field  $\Phi \in G$ ,  $\Phi^\dagger\Phi = 1$ , which transforms as  $\Phi \rightarrow U\Phi$  under  $G$ , and define new field variables:

$$h \rightarrow H = \Phi^\dagger h, \quad \psi \rightarrow \psi' = \Phi^\dagger \psi, \quad A_\mu \rightarrow A'_\mu = \Phi^\dagger \left( A_\mu + \frac{i}{g}\partial_\mu \right) \Phi. \quad (3)$$

In terms of these fields the Lagrangian (2) takes the form,

$$\mathcal{L} = -\frac{1}{2}\text{Tr}(F'_{\mu\nu}F'^{\mu\nu}) + i\bar{\psi}'\gamma^\mu(D'_\mu\psi') + i\bar{\chi}\gamma^\mu(\partial_\mu\chi) + [\lambda\bar{\psi}'H\chi + \text{h.c.}]. \quad (4)$$

It is important to note that (4) is not invariant under  $G$ . The field  $H$  is again non-dynamical, and can be viewed as a Lagrange multiplier which imposes the *gauge invariant* constraint

$$\bar{\psi}'\chi = 0. \quad (5)$$

Implementing this constraint explicitly in the functional integral<sup>3</sup> gives the equivalent Lagrangian

$$\mathcal{L}_{\text{equiv.}} = -\frac{1}{2}\text{Tr}(F_{\mu\nu}F^{\mu\nu}) + i\bar{\psi}\gamma^\mu(D_\mu\psi) + i\bar{\chi}\gamma^\mu(\partial_\mu\chi) + \frac{\lambda^2}{\mu^2}(\bar{\psi}\chi)(\bar{\chi}\psi), \quad (6)$$

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<sup>3</sup>This is equivalent to integrating out the auxiliary field  $H$  in the functional integral.

where<sup>4</sup>  $\mu^2 \rightarrow 0$ . This Lagrangian is equivalent to (4) and is manifestly gauge invariant under the transformations (1). Note that in the above manipulations no gauge fixing of the group  $G$  is involved; we merely changed the field variables which does not affect the symmetry properties of the functional integral measure. Therefore, the claim that the system is gauge invariant is an exact statement, i.e. it is true for the full quantum theory, not just for the classical one.

Observe that in going from equation (2) to equation (4) the correct implementation of the parameterization (3) does not introduce any new degrees of freedom. To see this more explicitly, we note that after the change of variables (3), the Lagrangian (4) is formally invariant under a new (different from the initial  $G$ )  $G'$  local gauge transformations:

$$G' : A'_\mu \rightarrow U' A_\mu U'^{-1} + \frac{i}{g} U' \partial_\mu U'^{-1}, \quad \psi' \rightarrow U' \psi', \quad \chi \rightarrow \chi \quad \text{and} \quad H \rightarrow U' H, \quad (7)$$

where  $U'(x) \in G'$ . The  $G'$  gauge freedom ensures that the number of degrees of freedom described by the new variables  $(A'_\mu, \Phi)$  is exactly equal to the number of degrees of freedom described by the old field variables  $A_\mu$ . The most illuminating choice is to take the Landau gauge, i.e.

$$\partial^\mu A'_\mu{}^a = 0, \quad (8)$$

where  $a = 1, 2, \dots, n = \dim(G')$ . Note that this is a gauge fixing condition with respect to the  $G'$  gauge symmetry, but merely a *gauge invariant* transversality condition on the vector field  $A'_\mu$  with respect to the initial  $G$  gauge symmetry. That is to say, the  $\Phi(x)$  field describes  $n$  longitudinal degrees of freedom of a vector field  $A_\mu(x)$ , not new degrees of freedom<sup>5</sup>. Therefore the lagrangians (2) and (4) are indeed fully equivalent.

Summarizing the lesson we have learned, in theories with explicit gauge breaking terms certain gauge invariant constraints might enforce the full gauge invariance. The degrees of freedom “missing” in the unconstrained gauge non-invariant theory must be non-dynamical at the classical level in order to generate the desired constraints through their equations of motion. These degrees of freedom become dynamical at the quantum level. Based on this observation, we now present a realistic model of electroweak symmetry breaking where the role of the Higgs boson is played by the dilaton field.

**The model** We would like to describe a scenario where the role of the Higgs boson is played by the dilaton, which, as was discussed above, must be non-dynamical at tree

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<sup>4</sup>We note by passing that the Lagrangian (6) describes the four-fermion Nambu–Jona-Lasinio model [6] with the four-fermion coupling  $G_{4f} = \frac{\lambda^2}{\mu^2}$ . The NJL model is often used in phenomenological studies of dynamical symmetry breaking.

<sup>5</sup>For an Abelian symmetry the condition (8) can be solved in closed form to express  $\Phi$  through  $A_\mu$ ,  $\log(\Phi) = ig \frac{\partial^\mu}{\square} A_\mu$ .

level. This is the case when the dilaton field  $\phi(x)$  couples to gravity with the conformal coupling  $\xi = 1/6$ ,

$$\mathcal{L}_{\text{grav}} = \sqrt{-\hat{g}} \left[ -\frac{\xi}{2} \phi^2 \hat{R} + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \mu^2 \phi^2 \right]. \quad (9)$$

In the above Lagrangian we have also included the dilaton mass term. This mass term explicitly violates the classical scale invariance of (9). We will have further comments on this Lagrangian later on.

Matter couples to gravity in the usual way through the diffeomorphism invariant Lagrangian,

$$\mathcal{L}_{\text{matter}} = \sqrt{-\hat{g}} \mathcal{L}_{\text{SM}}(\hat{g}_{\mu\nu}, \hat{F}) + \sqrt{-\hat{g}} \left[ \hat{Q}_L \hat{M} \hat{t}_R + \text{h.c.} \right]. \quad (10)$$

Here  $\mathcal{L}_{\text{SM}}$  denote a diffeomorphism covariant form of the Standard Model Lagrangian which involves all the Standard model fields, collectively denoted by  $\hat{F}$ , except the electroweak Higgs doublet.  $\mathcal{L}_{\text{SM}}$  is invariant under the  $SU(3) \times SU(2) \times U(1)_Y$  local gauge transformations, as well as under the classical scale transformations. We add  $SU(2) \times U(1)_Y$  violating fermionic mass terms, which also *explicitly* violate the scale invariance. For the sake of simplicity, we consider only the dominant<sup>6</sup> top-quark mass term in (10),  $\hat{M} = (m_t, 0)^T$ , where  $m_t$  is some mass parameter and  $\hat{Q}_L = (\hat{t}_L, \hat{b}_L)^T$  is the quark doublet. We regard the Lagrangian  $\mathcal{L} = \mathcal{L}_{\text{grav}} + \mathcal{L}_{\text{matter}}$  as a low-energy effective Lagrangian (in the Wilsonian sense) valid below some high energy scale  $\Lambda$ .

The metric  $\hat{g}_{\mu\nu}$  is taken in the so-called Jordan frame. From the particle physics standpoint however the Einstein frame must be regarded as the physical one<sup>7</sup>. We go to the Einstein frame by performing a Weyl rescaling of the metric  $g_{\mu\nu} = \Omega^2(x) \hat{g}_{\mu\nu}$ , and fields,  $F = \Omega^{S_F}(x) \hat{F}$  ( $S_F$  is the conformal weight of the field  $F$ : 0 for vector bosons,  $-1$  for scalar bosons and  $-3/2$  for fermions), taking  $\Omega(x)$  such that the rescaled dilaton field is equal to the reduced Plank mass  $M_P = 1/8\pi G_N \approx 2 \cdot 10^{18}$  GeV, i.e.  $\Omega(x) = \phi(x)/M_P$ . As a result of these rescalings the dilaton field is removed from the scale invariant part of the Lagrangian and appears only in the scale noninvariant mass terms. Thus we have:

$$\frac{1}{\sqrt{-g}} \mathcal{L} = -\frac{M_P^2}{2} R + \mathcal{L}_{\text{SM}}(g_{\mu\nu}, F) - \mu^2 h^T h + \left[ y_{\text{top}} \bar{Q}_L h t_R + \text{h.c.} \right], \quad (11)$$

where  $h = (\frac{M_P^2}{\sqrt{2}\phi(x)}, 0)^T$  and  $y_{\text{top}} = \sqrt{2}m_t/M_P$ . The Lagrangian (11) is phenomenologically equivalent to the fully gauge invariant NJL-type Lagrangian for the top mode Standard Model [7]. To observe this note that structurally (11) is the same as the

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<sup>6</sup>In view of neutrino see-saw masses, the neutrino Dirac mass terms might actually be dominant. See the comment below.

<sup>7</sup>The reason being that in the Jordan frame the dilaton has a kinetic mixing term with the scalar (trace) part of the graviton field, while in the Einstein frame provides diagonal basis for the kinetic terms.

Lagrangian in (2) and thus one may repeat the method above described. The classical equation of motion for  $h$  will enforce a constraint such that after integrating out  $h$  the full gauge invariance of the Lagrangian is manifest. The resulting form of the Lagrangian is equivalent to the NJL-type Lagrangian for the top mode Standard Model and thus describes the same phenomenology<sup>8</sup>.

The top mode Standard Model is known to suffer from some phenomenological problems; for example it prefers the top quark to be heavier than is experimentally observed. However simple variations of our proposed minimal model are possible. For example, instead of the top quark mass one could use a Dirac neutrino mass term  $m_D \bar{\nu}_L \nu_R$  as the dominate electroweak symmetry breaking mass scale (c.f. ref.[9]), and include the term  $\lambda \phi \bar{\nu}_R (\nu_R)^c$ . The resulting model would then be phenomenologically consistent and have the advantage of accommodating small neutrino masses via the see-saw mechanism. This modification would not alter our main point.

In conclusion, we have proposed an explicit model which enables the role of the Higgs boson to be played by the dilaton field. In the simplest version the resulting low energy effective theory is equivalent to the top mode standard model. Simple variations, accommodating neutrino masses, are also possible.

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<sup>8</sup>Also, the conformal coupling  $\xi = 1/6$  in eq. (9) can be justified as it appears as a stable infrared renormalization group fixed point in such type of models [8].

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